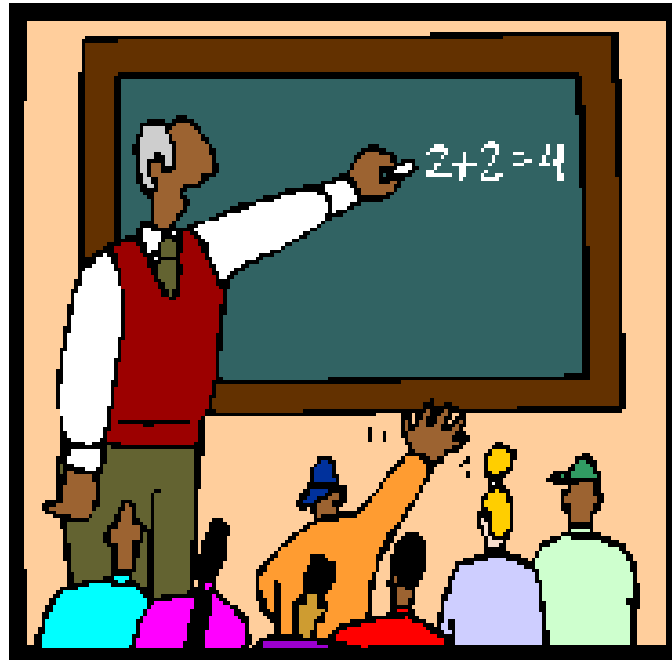


*“Using responsible gambling education to enhance our youth’s education – including life skills, controlling high risk behaviour, mathematics, statistics, core business principles, and HIV education”*

## **In this Workshop, Enoch the Maths Teacher teaches us about games of chance**



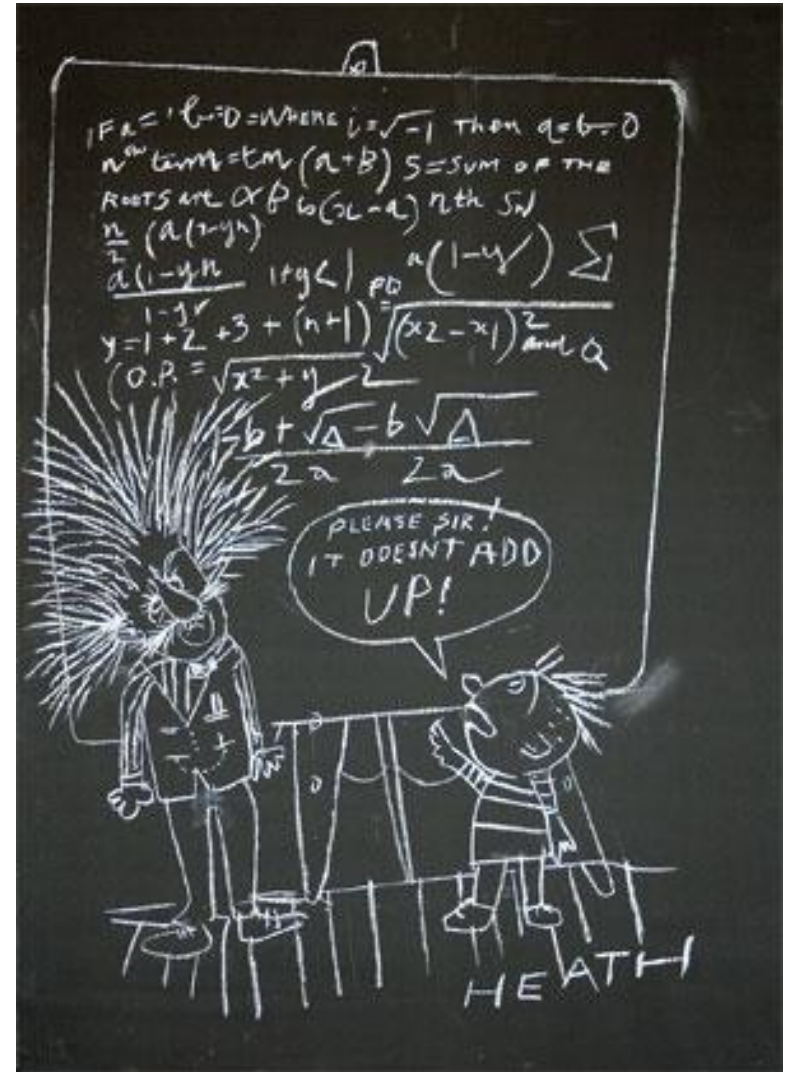
## This is Enoch....



Enoch is a Maths teacher, and we are going to tell a story of how Enoch taught his class about games of chance.

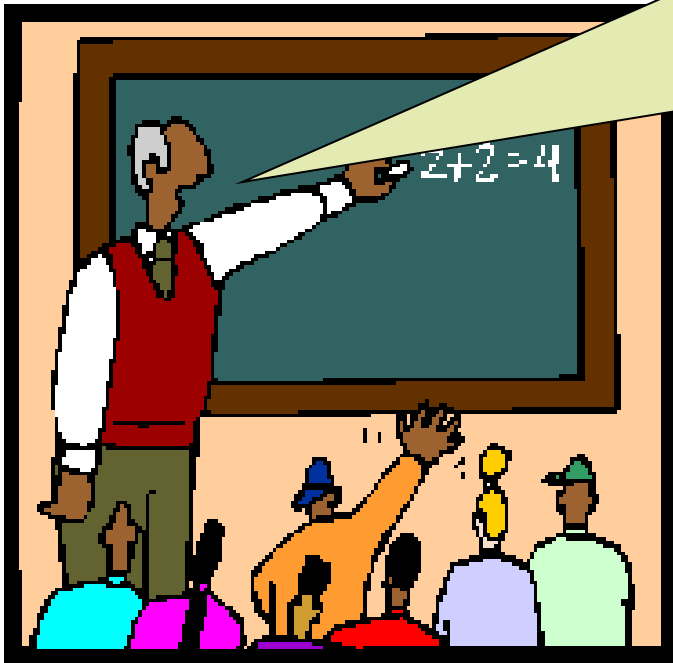
*“Using responsible gambling education to enhance our youth’s education – including life skills, controlling high risk behaviour, mathematics, statistics, core business principles, and HIV education”*

Following Enoch’s lesson, we don’t want this to be the case....



*“Using responsible gambling education to enhance our youth’s education – including life skills, controlling high risk behaviour, mathematics, statistics, core business principles, and HIV education”*

Before we can start class, we need to get to know some basics first. I am going to tell you about dice, and use that to teach you about probability and games of chance.



**The Basics : dice**

So, what's a dice?

Ok, for those wanting to correct my grammar, its :  
"What is a DIE?" – many *dice*, one *die*.

So, what is a die then?

Anyone who sighed or raised their eyebrows ...



Better not say it's a six sided cube.



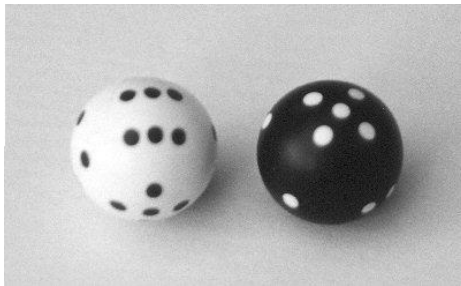
*“Using responsible gambling education to enhance our youth’s education – including life skills, controlling high risk behaviour, mathematics, statistics, core business principles, and HIV education”*

**The Basics : dice**

Well, a six sided cube with the numbers 1 to 6 is a die, but its just one kind of die.



Here are some other kinds.....



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**The Basics : dice**



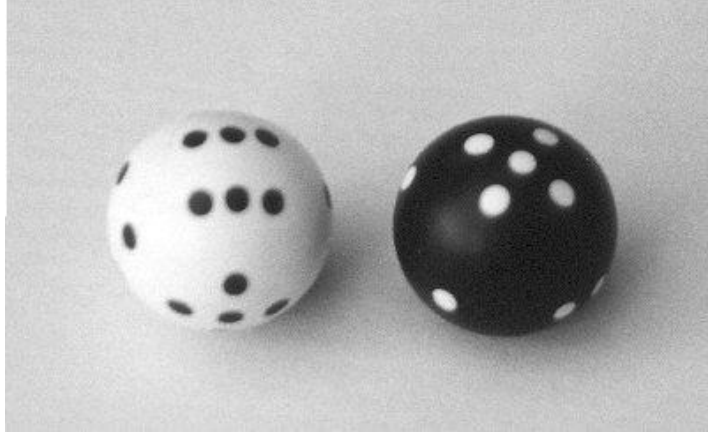
And for those still not convinced....



History of Dice >>>

*“Using responsible gambling education to enhance our youth’s education – including life skills, controlling high risk behaviour, mathematics, statistics, core business principles, and HIV education”*

**The Basics : dice**



These are a bit silly!!...when will they ever stop?

Imagine playing on an incline?





*"Using responsible gambling education to enhance our youth's education - including life skills, controlling high risk behaviour, mathematics, statistics, core business principles, and HIV education"*

We'll be concentrating on these kind...  
Who can tell me how many dice there are... ?

**The Basics : dice**



How many dice? >>>

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**The Basics : dice**



But to start off,  
we'll only  
consider one...

**The Basics : probability**



It has 6 sides – each side with a unique number. 1 through to 6.



.....So, let’s throw the die.

It comes to rest.

A number of unique outcomes exist....

Do you know firstly how many ?

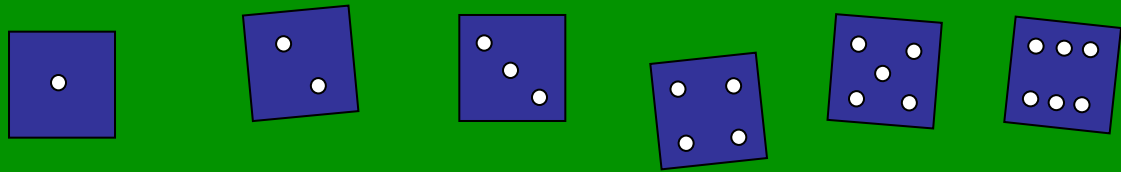
And secondly do you know what these are ?



**The Basics : probability**

The possible outcomes that may occur when throwing a die once are :

Get a 1; Get a 2; Get a 3; Get a 4; Get a 5; Get a 6



**The Basics : probability**

One can assume that the **chance** of the die landing on any of these numbers is the same.

This is also known as the *probability* (chance/likelihood)

Since there are 6 possible outcomes, the chance of one of them happening, is :

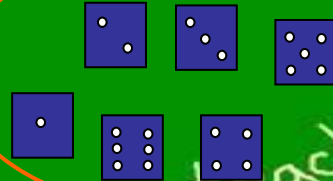
1 (the outcome) in 6 (the possibilities)



1

/

6



teacher →



**The Basics : probability**

Next, we’re going to talk about probability vs. odds as these 2 concepts often cause confusion.

We’ll begin by all agreeing that these are not good odds !!!

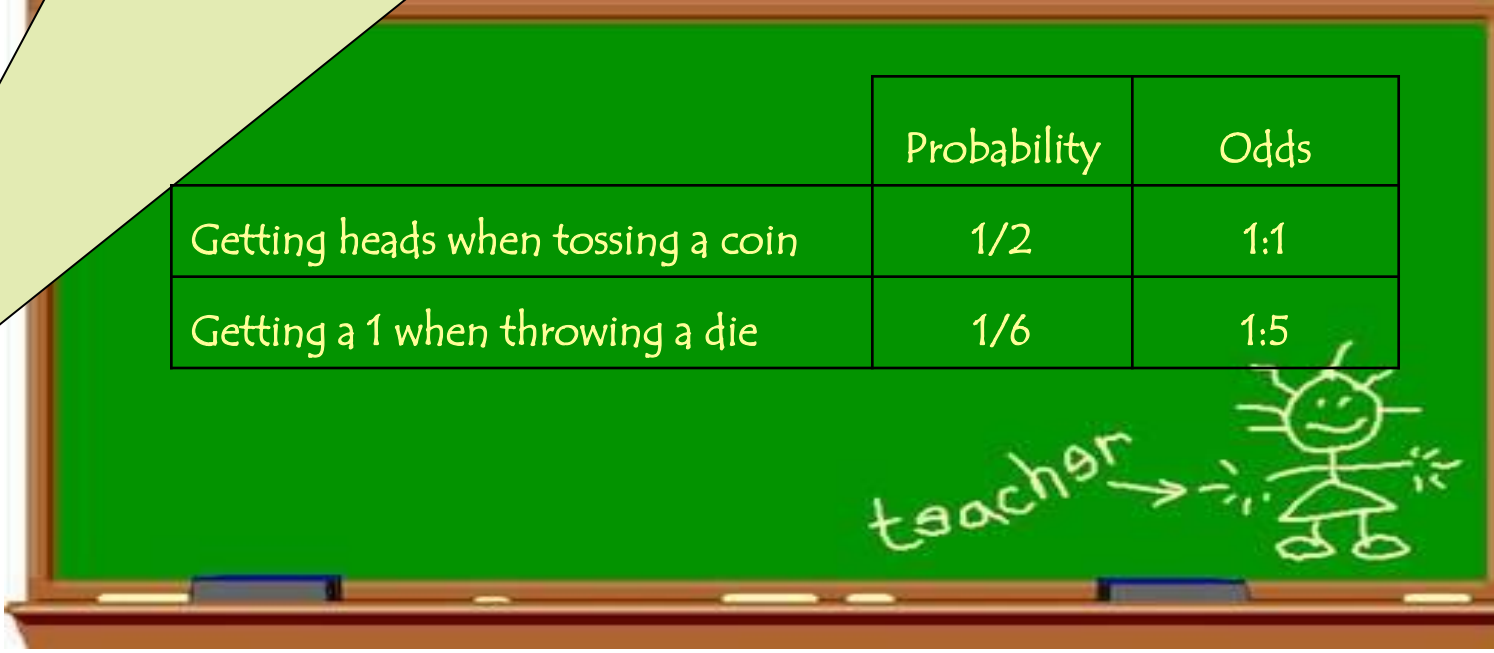


**The Basics : probability**

Probability is the chance of :  
 1 outcome vs. ALL possible outcomes

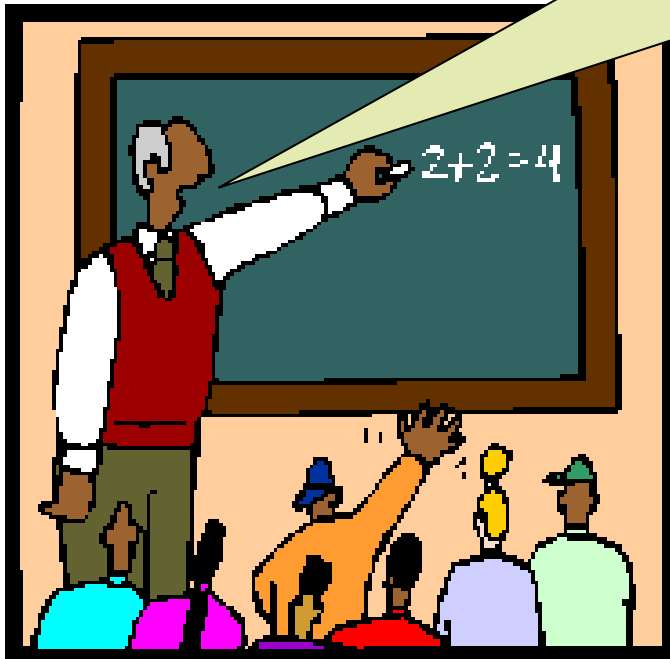
Odds describe the chance of :  
 Getting a desired outcome vs. the chance of NOT getting it.

	Probability	Odds
Getting heads when tossing a coin	1/2	1:1
Getting a 1 when throwing a die	1/6	1:5



Creating a game

Ok class, now that you know all about dice, and about probability, I think its time for us to create a make-believe betting game, using a die, in order for us to explore further concepts





**Creating a game**

For now, we going to assume that we are only going consider betting on a particular number.

Consider a game where people may bet on which number will come up in a particular throw of the die.

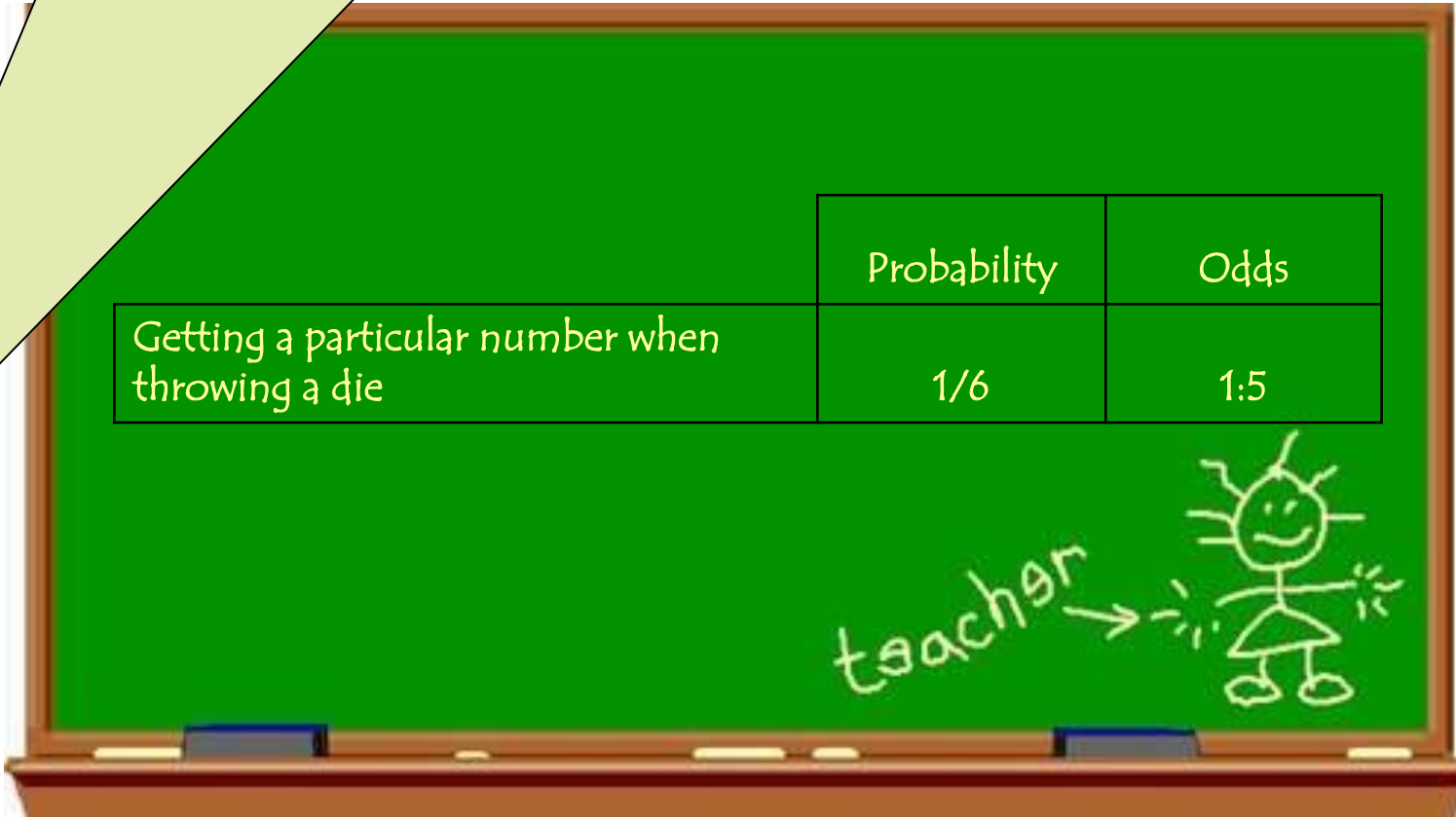


Creating a game

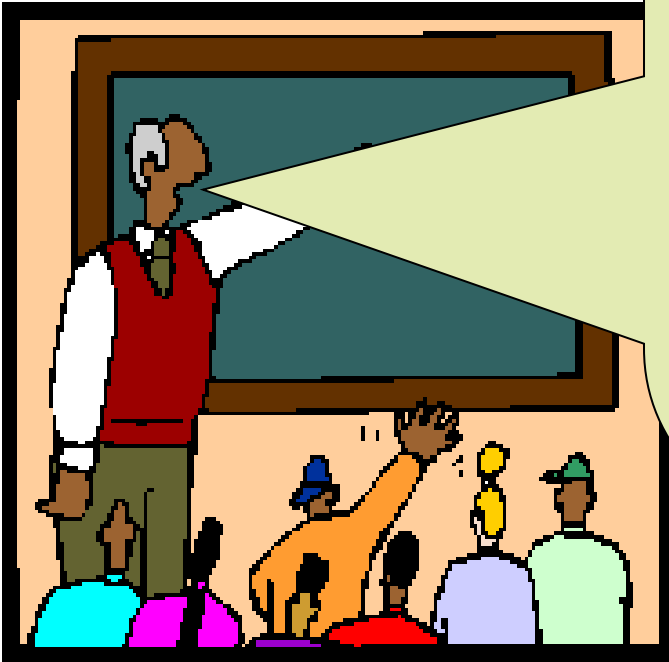
From previous discussions, we know what the odds and probability are of any particular number coming up when throwing a die :

Getting a particular number when throwing a die

Probability	Odds
$1/6$	1:5



Creating a game

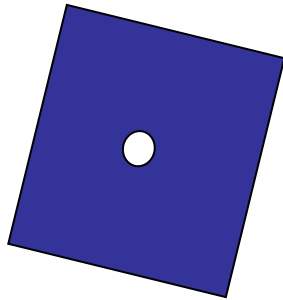


Right class, now I have an interesting question for you :

If we make the game such that, in the long run, by betting on various numbers, and losing some throws and winning others, a player neither wins nor loses money,

HOW MUCH MUST WE PAY OUT IF SOMEONE WINS, FOR THE GAME TO BE A 'BREAK EVEN' GAME IN THE LONG RUN ?

**Creating a game**



To help you, go through the following questions :

Think of betting R1 on number 1 coming up on any particular throw.

Think about how many throws (in theory) would be needed to get at least one number 1 coming up?

How many of those throws would have been losing bets?

How much did you lose overall?

How much do you need to get paid on your winning bet to "make up" for your losses?



**Creating a game**

In theory number 1 will come up once in six throw. One will thus have made six bets of R1. A total of R6 “staked” or bet.

5 of the bets will be losing bets, hence R5 will be lost.

Only one bet will be a winning bet (remember, in theory), and thus that one winning bet, for the game to allow players to break even will need to make up or cover the losses of the 5 winning bets. The winning bet will therefore have to pay out R5.

The R1 bet on the winning bet is obviously kept by the player !

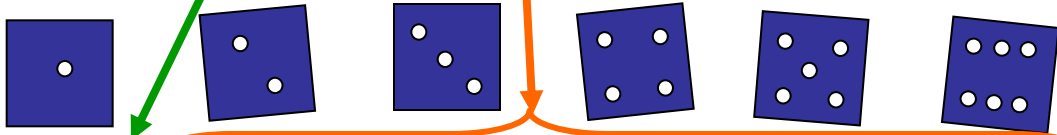


**Creating a game**

Below is a diagram showing the required payout for the game to break even. Remember, the player is betting on number 1 coming up. 6 games are played, with a different number coming up after each game.

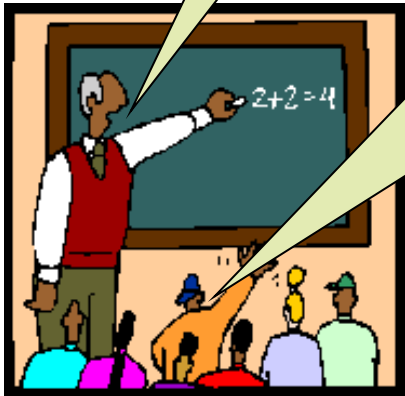
This R1 is kept by the player on the winning bet.

These are all lost.



Game no	1	2	3	4	5	6	
Amt bet	R1	R1	R1	R1	R1	R1	R6
Amt lost	-	R1	R1	R1	R1	R1	R5
Amt won	R5	-	-	-	-	-	R5

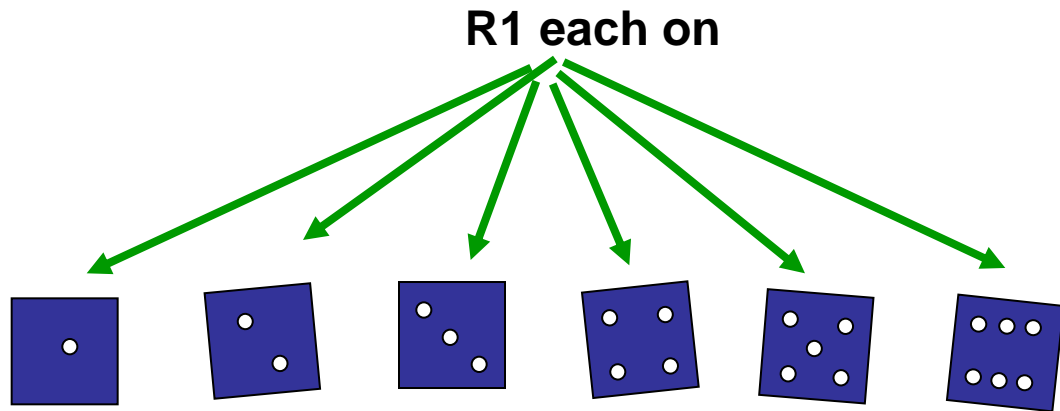
Creating a game



Yes Jim?

Sir, I was just thinking, wouldn't it be good test of a break even game if one bet on **all the numbers**, and after each play you get back exactly what you bet or stake ?

R6 staked on each throw. 5 will lose a R1 each (total R5), but one will win R5



Creating a game

Excellent point Jim. Very much so. This is also known as the “fair game test”. This test says that if you bet on every possible outcome you should neither win nor lose.



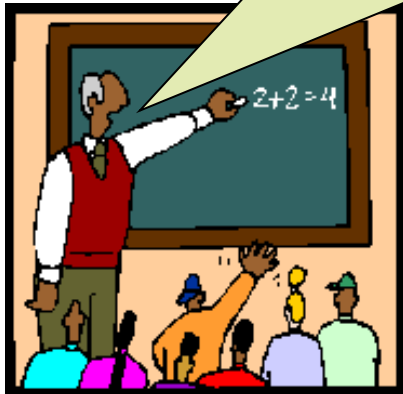
The fair game test is one of the ways to determine whether a game is ‘fair’. Strictly speaking, statisticians call it a “statistically fair game” when a player has an ‘expected’ return of 100% over time. Over time, they are not expected to win or lose anything.



Creating a game

In the long run then, no money will be won or lost by player or 'host' of the game (e.g. casino). Often you will hear people talking about 'in the long run' when talking about probability.

So how long do you have to wait !!!?  
(click the link to understand convergence)



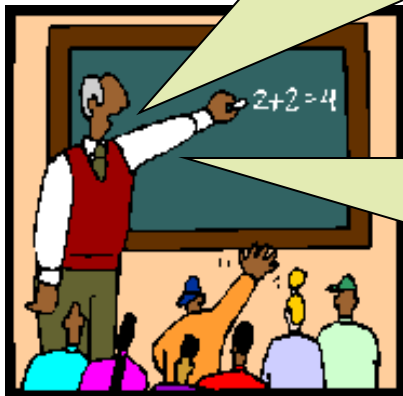
Convergence >>>

Creating a game

## Is it a good idea to offer a statistically fair game?

The answer to this may seem simple – but let’s change the question to: *Do you want to make money ?*

- If **yes**, then its not a good idea to offer a statistically fair game
- If **no**, then it is a good idea to offer a statistically fair game



The above may seem correct, but overheads, or running costs of any business require it to at least make some money to cover these costs, even if it makes no profit....(click to learn more)

**Creating a game with a house advantage**



Enoch suddenly has a great idea...

And shares it with his class.....



I've just had a great idea class. How about creating a variation of the game like the one we've just worked on, for visitors to play at the school fete, in order to make money for the school !?



We’ve done a great job at working out the payouts for a fair game.

For the stall to win money for the school though, the payouts need to be tipped in our favour, and against the players.

Let’s try and determine how we should tip the odds.....

Any ideas?

Remember, we previously paid out R5 for a winning bet where the player bet R1 on that number.

Let’s ask some basic questions :

What would happen if we paid more than R5 ?

What would happen if we paid R2 ?



If we paid more than R5, we would be tipping it in the *players* favour and the stall would *lose* money over time.

If we only paid R2 each time someone won, while they lost R5 on the other 5 bets, they may feel that it wasn’t “worth it” and no one would play our game!

**Creating a game with a house advantage**

The advantage that we want to give the stall is called the “house advantage”.

Let’s decide on paying out a little less than the usual R5 per win (in the fair game) and pay out R4.50 instead.

We now need to determine what the “house advantage” will be in this case in %.



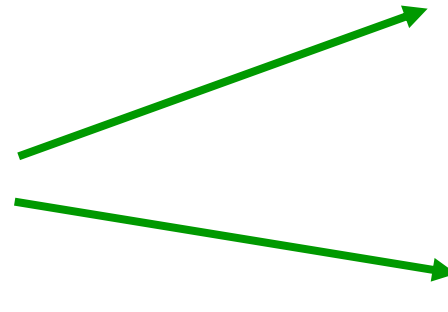
Fair game  
winnings



New game  
winnings



House  
advantage



**Creating a game with a house advantage**

A house advantage simply refers to the fact that, in the long run, the house or casino has a higher expectation of winning than the player.



Have a look at the board on the next page to see how we work out “house advantage”



## Working out house advantage

1. Bet the same bet on all possible outcomes - R1 on each number
2. Determine total outlay - also called 'handle' or 'total amount staked' = R6
3. For a statistically fair game, pay R5 (+R1 of original bet) for the winning number
4. For the house advantage, we will keep 50c, and only pay R4,50 + R1 original bet for a win.
5. Win percentage (win %) is
  - ◇  $\text{Win}\% = \text{win} / \text{handle} \times 100$
  - ◇  $\text{Win}\% = (5.5 / 6) \times 100$
  - ◇  $\text{Win}\% = 91.67\%$  (to 2 decimal places)





**Working out house advantage continued...**

$$\text{Win Percentage} = \frac{\text{Payout (including original stake)}}{\text{Stake (on all possible outcomes)}} \times 100$$

House advantage is “what’s left over” =  $100 - \text{Win Percentage}$

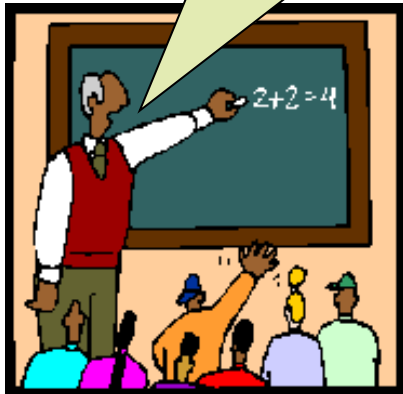
So for this game, the win % is 91.67% and the house advantage is 8.33%.



**Limiting Risk in Betting Games**

Ok kids, this seems like a great idea. Our house advantage is set at about 8%.

Now we need to think if there is any risk to our plans to make the school money.



The biggest, and most dangerous risk is if someone comes along and bets a huge amount (say R10,000) on one number. If that number comes up, the stall would have to pay the player R45,000. The stall certainly and possibly the school could face financial ruin. Our great Maths Teacher Enoch may well lose his job....



[More about Risk >>>](#)

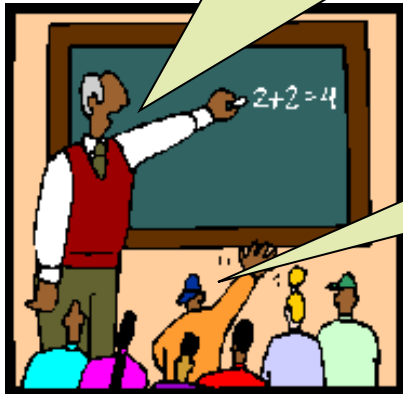
**Limiting Risk in Betting Games**

The danger of a high roller betting one large amount on a single number is a big concern.

Who can think of a way to limit that risk?

Yes Jimmy ?

What about limiting how much someone is allowed to bet per number per game ?



**Limiting Risk in Betting Games**

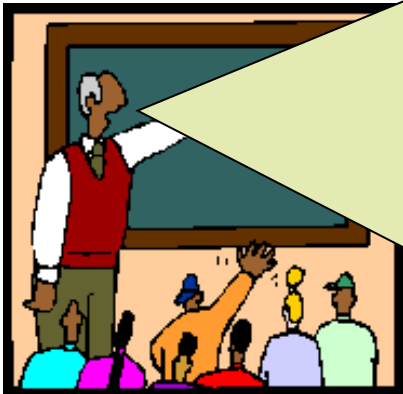


Very good suggestion.

Limiting the stake (the amount someone can bet on a particular number), and because we know that the longer the game is played the more likely we are to turn a profit, it will reduce the risk.

If we limit the bet to say R5, then our maximum payout (excluding the original bet) for any particular person and their bet is  $R5 \times 4.5 = R22.50$ . If we include the original bet the payout is R27.50.

**Limiting Risk in Betting Games**



I am feeling good about our stall kids.

- We have a house advantage.
- We’re limiting bet size.

What else can we do?

Well we could try and encourage people to make “offsetting bets”. So if we had a lot of people betting R5 on the number 3 and we could encourage people to bet an equally large amount on some other number we could cancel or nullify our potential loss on 3 since only one number can win.

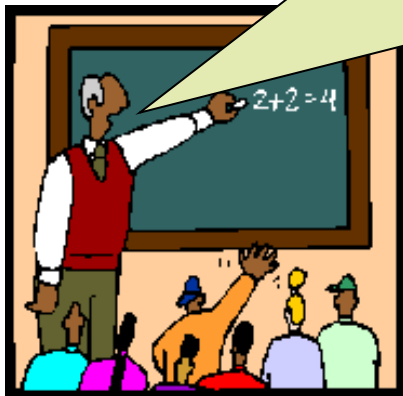
**Limiting Risk in Betting Games**

So, what are the 3 things that professional gambling organisations (like casinos) try and do to limit risk.

1. Set the payout odds so that they give themselves a house advantage.

2. Limit the size of any particular bet

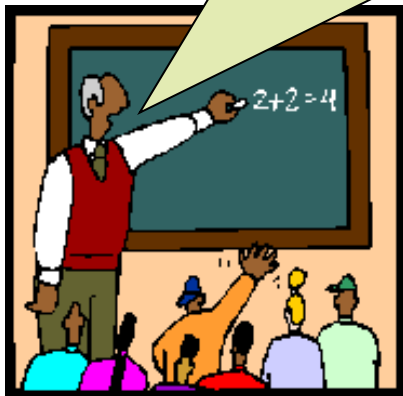
3. Encourage, where possible, offsetting bets. Generally simply having as large a number of people playing a particular game will result in offsetting effects.



**The perspective of the player**

People play games like the die game above for the excitement or thrill of having a chance of winning something.

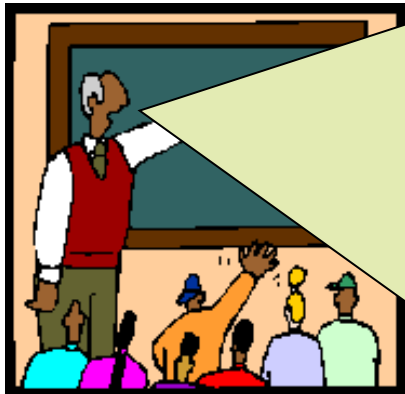
They are prepared to accept their expected loss (the house’s advantage) because the act of playing and the associated excitement is entertaining.



**The perspective of the player**



**vs.**



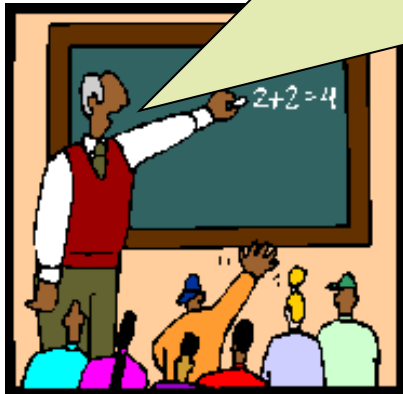
As we saw above a gambling establishment would always prefer an even spread of bets because they are then least at risk of having a large loss.

The player him/her self, being the person who does the betting, can also manage the profile of what we generally call player risk.

They can, for example, bet the maximum bet on a single number or they might want to spread their risks and put a small amount of money on say three numbers – this gives them a larger chance of winning something (what casinos call the Hit percentage) but less chance of winning a large amount.



**The perspective of the player**



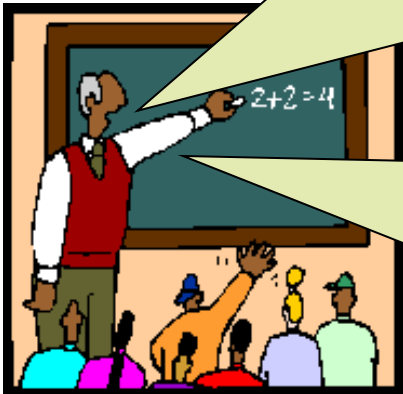
It worries me that although the higher risk-taking players who put “all their eggs in one basket” and just bet on a single number are catered for, but that there might be some risk averse players (players who would rather win something pretty frequently) who won’t be satisfied with a one in 6 hit percentage.

I think that for those players we need to create additional categories of betting that have a higher hit percentage...

**The perspective of the player**

Let’s have 4 extra categories of bets at our stall to accommodate risk averse players:

- a) The numbers  $\{1, 2, 3\}$  – [numbers less than or equal 3]
- b) The numbers  $\{4, 5, 6\}$  – [numbers greater than or equal 4]
- c) Odd numbers  $\{1, 3, 5\}$
- d) Even numbers  $\{2, 4, 6\}$



The hit percentage is far higher than the 1 in 6  
16.6% of betting on one number.  
With these categories, the hit % will be 50%.  
On average one out of every 2 bets will pay out.

**The perspective of the player**

If the game were fair Enoch knows that for each of the bets above he would pay out the amount staked (plus the original bet), but what does he pay out if he wants the same house advantage that he had with the single number bets.

The way to work it out is as follows: Let’s consider the odd and even bets. Say someone bets on both so that the amount staked on all possible outcomes is R2. If the game were fair they would simply always get their money back (R2) . If we run a house advantage of  $\frac{1}{12}$  (8.33%) or equivalently a win fraction of  $\frac{11}{12}$ .



Keeping the same win fraction we must have

$$\frac{\text{Payout (including original stake)}}{\text{Stake (on all possible outcomes)}} = \frac{11}{12}$$

I think you can work out that the Payout must be  
 $R\ 2 * 11/12 = R1.83c$  (including the original bet)

So we must pay out 83c on the winning even/odd bets  
and similarly on the {1, 2, 3} and {4, 5, 6} bets to  
maintain a consistent house advantage (Win%).



**The perspective of the player**

We need to keep a lot of change to deal with all the 83c payouts. This is a real pain.

I think we should just make it 80c – you can probably see that gives a house advantage of exactly **10%**.

This results in numbers which are nice, and rounded.

How will this affect the other single number bets?! Will these still be rounded off? We’ll we work it out in the same way:

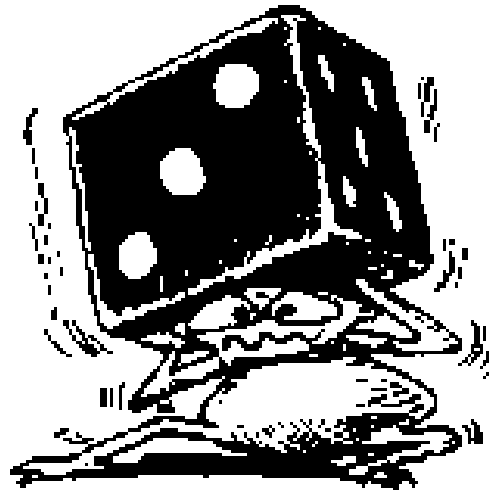
The stake for all possible outcomes is 6 (each of the 6 numbers). So the payout must be R5.40 – whew, not too bad and no strange decimals.

So for each winning number Enoch pays out R5.40 (including the original bet) – or R4.40 excluding the original bet.



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Enoch goes home to test the game out  
on his friend Monica



Monica thinks the payoffs are wrong. “If I put R1 in evens and I get an even then I must win a R1 says Monica, not 80c. Enoch tries to explain about house advantage but Monica finds this confusing.”

So Enoch gets another idea. He carves a 7-sided die from a piece of wood; it has a top and a bottom and 5 sides around its middle. It has the numbers 1 through to 6 on it as well as 0. Then he says he will use his 7-sided die at the fair but will call 0 a dead number. That is, it won’t be included in the odds or even. You can bet on it and it will just pay what the other numbers pay but it won’t be included in the odds or evens or other grouped bets.

Enoch decides to pay R5 on the numbers (including 0) for a R1 bet (R6 including the bet) and R1 on the odds and even, like Monica wanted. If {0} comes up then the bets on {0} get paid out but bets on the odds and evens and the {1, 2, 3} and {4, 5, 6} don’t get paid out.

This seems much simpler in terms of handling the money and Monica thinks this is a much better arrangement.

Enoch explains this to his class the next day...



...so class, now that we are changing the payouts we need to work out the new win percentage :

$$\text{Win Percentage} = \frac{6}{7} * 100 = 85.71\% \text{ (2 decimals)}$$

Using the win %, who knows what the house advantage will now be?

Yes, house advantage is 14.29% (100-85.71) – quite a lot more than before!



Can we check that paying out R1 on the odds and even also gives the same house advantage....

This is a bit trickier. Lets consider playing the game 7 times. We would expect to get one of each of the seven numbers. Moreover, say each time we bet R1 on (each of) odds and evens and R1 on {0}, so that each time we had to win something.

The 7 plays cost us R21; our winnings (including the stake) are R2, 3 times on the odds (R6 in total), R2, 3 times on the evens (R6 in total) and R6 once on the {0}, to total R18.

The Win fraction is  $18/21 = 6/7$  exactly the same fraction as with the single number payoffs. Hence the house advantage remains 14.29% for all the potential bets.



Enoch is really happy – a simple arrangement with no fractions and with payoffs that everyone seems to find more intuitive. Including a dead number has fixed his problems – he’s ready for launch. The only remaining problem is that 14.29% seems a pretty steep house advantage – Enoch could expect in the longer term to be getting a return of 14.29% on the money bet each time the game is played. Investors think 14.29% is a good return for a whole year! Maybe no-one will play the game after all.

Enoch starts thinking ... yet again :-)

He remembers a game called Roulette and that he has an old Roulette wheel. He digs it out and sees it has 36 red and black numbers ( 1 through to 36) as well as a green {0}. It works on the same ideas he had developed for the die game. A dead 0 !! but because there are a lot more numbers (36 rather than 6 on a die) the house advantage is much more reasonable.

The parallels between Roulette and his 7-sided die with the dead number are extraordinary. Roulette also has only one dead number (green {0}) and pays out intuitively – you receive R2 (per R1 bet) when you win on the odds and evens (as well as the equally split reds and blacks), you receive R36 (per R1 bet) when you win on an individual number but because there are a lot more numbers the win percentage is much less. Moreover, there are a number of other betting options – you can bet on {1 – 12}; {13 – 24} or {25 – 36}; this pays out R2 (excluding the original bet) on a R1 bet.

Enoch starts reading about Roulette; he discovers the version he has is the original European Roulette that they use at the famous Monte Carlo casino. In the USA they use a different Roulette wheel with a single 0 AND a double 0. This has a much larger house advantage – very close to double the European version.